Learning while Bargaining: Experimentation and Coasean Dynamics

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In **bargaining** theory and practice, **outside options** play an important role.

In most markets or bargaining situations, outside options

- **May or may not exist**;
- **May take time to arrive**.

Thus, parties invest time and resources (**search**) to **learn** what their best options are

- Labor, housing, and financial markets, M&A negotiations, etc.

How do bargaining relationships evolve in markets with search and learning?

Setting

Bilateral bargaining with one-sided incomplete information

- An indivisible good;
- One uninformed seller, one buyer with private valuation for the good;
- The seller makes price offers, the buyer accepts or rejects;
- Infinite horizon, discounting, frequent offers, no commitment;
- Continuous time.

Setting

Uncertainty about **whether and when** superior outside options are available

- Good match: outside options are not available;
- Bad match: Poisson arrivals of outside options (on either or both sides);
- Common prior, public arrivals.

Gains from trade are ex ante uncertain.

Common learning

- If trade is delayed, info about outside options comes to light, uncertainty unravels.

First best efficiency: **optimal delay** (not immediate agreement)

- Costs and benefits of experimentation must balance.

Questions

How do parties bargain in the face of market uncertainty and its unraveling?

How do learning about outside opportunities and screening of private information interact?

- Timing of transactions and efficiency properties of equilibria?
- Do parties gather too much or too little information?
- Price dynamics and surplus division?

Preview

Delay is always present

- Different (groups of) types trade at different times;
- Trade occurs over time because of search and learning while bargaining.

Efficiency *vs* inefficiency depends on IPV *vs* IV

- With IPV, delay is efficient (knife-edge case);
- With IV, trade is inefficient: **excessive delay** as well as **excessive hurry**
  - It depends on the **sign of the intrapersonal externality among seller's multiple selves**.

Seller's bargaining power depends on a **market clearing condition**

- The market clears in finite time iff the market clearing price is positive;
- If the market clearing price is positive, price offers and the seller's payoff are higher than competitive.

Bargaining externalities and market unraveling.

Equilibrium in closed-form.

Literature: A First Look

Bargaining with One-Sided Incomplete Information and Durable Goods Monopoly

- **IPV**: Coase (1972); Stokey (1981); Bulow (1982); Fudenberg, Levine, and Tirole (1985); Gul, Sonnenschein, and Wilson (1986); Ausubel and Deneckere (1989).
- **IV**: Evans (1989); Vincent (1989); Deneckere and Liang (2006).

Bargaining in Changing Environments (or Outside the Void)

- Inderst (2008); Fuchs and Skrzypacz (2010); Huang and Li (2013); Board and Pycia (2014); Ortner (2017); Daley and Green (2020); Chaves (2020).

Surveys

- Ausubel, Cramton, and Deneckere (2001); Fuchs and Skrzypacz (2020).

Model

Players and Values

Players: a **seller** and a **buyer**.

The seller owns an **indivisible good** (or asset) valued at 0.

The buyer has **private valuation** $v \in [0, 1]$ for the good (buyer type)

- v is distributed according to a c.d.f. F;
- F is an atomless distribution with full support and density f.

Time, Actions, and Payoffs

Continuous time, infinite horizon: $t \in [0, +\infty)$

- Players are long-lived and risk-neutral expected utility maximizers;
- Common discount rate: $r > 0$.

No commitment power

- At every t, the seller offers a **price** p_t to the buyer, who accepts or rejects.

If the buyer accepts p_t, trade is executed and the game ends with **payoffs**

- For the seller: $e^{-rt} p_t$;
- For the buyer: $e^{-rt}(v - p_t)$.

Market

The **market** is of type $m \in \{0, 1\}$

- $m = 1$: an **event publicly occurs** according to a Poisson process with intensity $\lambda > 0$;
- $m = 0$: no event ever occurs.

Common prior $\mu^0 \in (0, 1)$ on $m = 1$ at $t = 0$.

The **event** is an **outside option** arriving for either or both players

- **Uncertainty** about **whether and when** outside options are available.

Market

The arrival of an event at time t ends the game with **payoffs**

- For the seller: $e^{-rt}O^S(v)$;
- For the buyer: $e^{-rt}O^B(v)$.

Players have

- **Independent Private Values (IPV)** if $O^S(v) = O^S$ for all $v \in [0,1]$ and some $O^S \geq 0$;
- **Interdependent Values (IV)** otherwise.

Uncertain Gains From Trade

Gains from trade are ex ante uncertain: for a positive-measure subset of $[0,1]$,

$$\frac{\mu^0\lambda}{\mu^0\lambda + r} O(v) > v, \qquad \text{where} \qquad O(v) := O^S(v) + O^B(v).$$

- If $m = 1$, better deals exist, at least in terms of joint surplus to share.

Common Learning

Since the event is public, players share a **common belief** μ_t on $m = 1$ at every t

- Before an arrival or an agreement, the common belief drifts downwards according to

$$\dot{\mu}_t = -\lambda \mu_t (1 - \mu_t), \qquad \mu_0 = \mu^0.$$

No news is bad news for the joint surplus

- Not necessarily from each players' individual viewpoint.

First Best

Option value of market experimentation

- If trade is delayed, info about outside options comes to light;
- Uncertainty about gains from trade unravels because of learning.

First-Best Efficiency

- At belief μ trade occurs with all buyer types v for which

$$\underbrace{\mu\lambda[O(v) - v]}_{\text{benefit of experimentation}} = \underbrace{rv}_{\text{cost of experimentation}}.$$

- Some **delay** is bilaterally efficient.

Other Assumptions

1. $O^S(v)$ and $O^B(v)$ are non-negative differentiable functions.
2. $v - O^B(v)$ is increasing on $[0, 1]$
   - Higher buyer types are more eager to trade.
3. $O^S(v)$ is either a constant or a monotone function on $[0, 1]$
   - Either the seller's payoff from the outside option does not depend on v,
   - Or the seller prefers high/low v's.

Heuristic Timeline within "Period" $[t, t + \mathrm{d}t)$

1. The period begins with a common belief μ_t on $m = 1$.
2. The event occurs with subjective probability $\mu_t \lambda \mathrm{d}t$
   - The game ends and players collect payoffs.
3. With probability $1 - \mu_t \lambda \mathrm{d}t$ no event occurs and the seller offers price p_t
   - If the buyer accepts, the game ends and players collect payoffs;
   - If the buyer rejects, players update beliefs and move to the next "period".

Continuous Time

No commitment power and no frictions in the protocol

- Except for those one intends to model.

Tractability

- Optimality conditions and equilibrium strategies described by HJBEs, PDEs, ODEs;
- Closed-form solutions;
- Comparative statics, predictions for empirical studies and applied research.

The Event

Not a model of what the event stands for or the strategic interaction it gives rise to

- Any breakthrough in some search activity parties engage in during the bargaining process.

Replace the event with the payoffs of the continuation game it gives rise to

- The few assumptions on $O^S(v)$ and $O^B(v)$ allow to capture a variety of applications
- E.g., **arrival of new traders**.

Examples

Sellers' market: $O^B(v) = 0$ for all $v \in [0,1]$

- Outside options, if existing, only benefit the seller;
- E.g., arrival of short-lived buyer with valuation $V = 1$ or coming to offer price $V = 1$.

Buyers' market: $O^S(v) = 0$ for all $v \in [0,1]$

- Outside options, if existing, only benefit the buyer;
- E.g., arrival of a short-lived seller with an upgraded version of the good.

Examples

General market

- The event benefits the buyer with prob. $\alpha \in (0,1)$ and the seller with prob. $1-\alpha$.
- The event alters both parties' payoffs at the same time
  - Let $v \sim U[0,1]$
  - Event: arrival of a new buyer with valuation $\tilde{v}$ independent of v and, for some $\beta \in (0, 1/2)$,

$$\tilde{v} \sim \begin{cases} U[0,1] & \text{with prob. } \beta \\ U[2,3] & \text{with prob. } 1-\beta \end{cases}$$

  - First price auction upon arrival
  - Original buyer bids $b(v) = v/2$; new buyer bids $\tilde{b}(\tilde{v}) = \tilde{v}/2$ if $\tilde{v} \in [0,1]$ and $\tilde{b}(\tilde{v}) = 1$ if $\tilde{v} \in [2,3]$
  - Payoffs upon arrival

$$O^S(v) = \beta \int_0^1 \frac{\max\{x, v\}}{2} \mathrm{d}x + (1-\beta) = \frac{4 - \beta(3 - v^2)}{4}$$

and

$$O^B(v) = \beta v \left(v - \frac{v}{2} \right) = \frac{\beta v^2}{2}.$$

Equilibrium Notion

Equilibrium Notion

Ad hoc equilibrium notion for the continuous-time bargaining game

- Ortner (2017); Daley and Green (2020); Chaves (2020).

Equilibrium Notion

Buyer's optimal stopping problem

- Given the arrival process and a price path, when to accept the offer?
- **Skimming property**: optimal stopping times are decreasing in type.
- Thus, types remaining at any time are a right-truncation of the original type distribution.

Natural state variables for Markov strategies

- k: cutoff type defining the current right-truncation of the type distribution;
- μ: current belief on the type of the market environment.

To any given price path in the range of the buyer's reservation price strategy,

- There corresponds a path of realized cutoff types.

Instead of prices, **seller chooses a path for cutoff types**—how quickly to screen buyer types.

Equilibrium Objects

Equilibrium objects are given by the pair

$$\left\{\left\{\left(K_t^k\right)_{t\in[0,+\infty)}, k \in [0,1]\right\}, P(\cdot,\cdot)\right\}.$$

$\left\{\left(K_t^k\right)_{t\in[0,+\infty)}, k \in [0,1]\right\}$ is a collection of paths $t \mapsto K_t^k$ for cutoff types, one for each initial cutoff $K_{0^-} = k$

- Chosen by the seller;
- $\left(K_t\right)_{t\geq 0}$ is non-increasing, measurable, right-continuous.

For all $(k,\mu) \in [0,1] \times [\mu^0, 1)$, $P(k,\mu)$ is simultaneously

- Type $v = k$'s **reservation price strategy**,
- The **price offered** in state (k,μ).

Markov Equilibrium

The pair $\left\{\left\{\left(K_t^k\right)_{t\in[0,+\infty)}, k\in[0,1]\right\}, P(\cdot,\cdot)\right\}$ is a **Markov Equilibrium** if the following holds.

1. Buyer optimality

- For all $v\in[0,1]$, $k\in[0,1]$, and $\mu\in[\mu^0,0)$, $P(v,\mu)$ is an optimal reservation price strategy for v given
  - The arrival process;
  - The law of motion of $\left(K_t^k\right)_{t\in[0,+\infty)}$;
  - Future prices given by $P(K_t^k,\mu)$

2. Seller optimality

- For all $k\in[0,1]$ and $\mu\in[\mu^0,0)$, $\left(K_t^k\right)_{t\in[0,+\infty)}$ is a seller-optimal path for cutoff types.

Regular Markov Equilibrium

Restrict the seller's strategy space to paths for cutoff types $\left(K_t^k\right)_{t\in[0,+\infty)}$ such that

$$\boldsymbol{K_t^k} = \boldsymbol{K_t}^{k,\text{abs}} + \boldsymbol{K_t}^{k,\text{step}}$$

- $K_t^{k,\text{abs}}$ is an absolutely continuous function;
- $K_t^{k,\text{step}}$ is a step function with finitely many jumps.

Such strategies capture

- **Smooth trade**: type by type;
- **Silent trade**: time intervals with no trade;
- Finitely many **atoms** of trade.

Regular Markov Equilibrium

Since $\left(K_t^k\right)_{t\in[0,+\infty)}$ is monotone, it has Lebesgue decomposition

$$\boldsymbol{K_t^k} = \boldsymbol{K_t^{k,\text{abs}}} + \boldsymbol{K_t^{k,\text{jump}}} + \boldsymbol{K^{k,\text{sing}}},$$

- $K_t^{k,\text{abs}}$ is an absolutely continuous function;
- $K_t^{k,\text{jump}}$ is a piece-wise constant jump function;
- $K^{k,\text{sing}}$ is a singular continuous function (non-constant, continuous, with first derivative equal to zero a.e.).

Restriction

- Jumps are rare, the continuous part of $\left(K_t^k\right)_{t\in[0,+\infty)}$ is sufficiently smooth.

Regular Markov Equilibrium

Regular Markov Equilibrium (RME): Markov eqbm. when the seller's strategy space is restricted as above.

Proposition

There exists a unique RME.

By construction and verification

- Necessary conditions identify a unique candidate RME, sufficiency via a verification argument.

Equilibrium Characterization and Main Result

Preliminary Observations

The market cannot clear in finite time

- This would have to occur at zero or negative price;
- Dominated (for the seller) by waiting for an arrival or not trading.

Thus, there are no instant-trade equilibria and no final atom of trade

- There must be a final absorbing region with smooth trade

$$\dot{K}_t \in (-\infty, 0] \quad \text{for all} \quad t \geq \overline{t} \geq 0.$$

Preliminary Observations

No silent periods after trade begins

- During a silent period, the seller's (Coasean) desire to speed up trade must be absent;
- However, because of learning, over a silent period

  Either seller's opportunity cost of trading or cutoff type's willingness to pay (or both) must change;
- If $k_{\overline{t}} < 1$ for some $\overline{t} \geq 0$, then

$$\dot{K}_t \in (-\infty, 0) \quad \text{for all } t > \overline{t}.$$

Seller's Problem

At state (k,μ) with smooth trade, the seller's value function must satisfy the HJB equation

$$\underbrace{rS(k,\mu)}_{\substack{\text{expected payoff}\\ \text{in flow terms}}} = \sup_{\dot K\in(-\infty,0]} \Bigg\{ \underbrace{\mu\lambda}_{\substack{\text{flow prob.}\\ \text{of arrival}}} \underbrace{\left[\underline{O}^S(k) - S(k,\mu)\right]}_{\substack{\text{if arrival, game ends,}\\ \text{seller earns } \underline{O}^S(k) \text{ and forgoes } S}} + \underbrace{\left[P(k,\mu) - S(k,\mu)\right]}_{\substack{\text{if buyer accepts } P\text{, game ends,}\\ \text{seller earns } P \text{ and forgoes } S}} \underbrace{\frac{f(k)}{F(k)}(-\dot K)}_{\text{flow prob. of trade}}$$

$$+ \underbrace{\frac{\partial}{\partial k}S(k,\mu)\dot K + \frac{\partial}{\partial \mu}S(k,\mu)\dot\mu}_{\substack{\text{change in continuation payoff}\\ \text{if no arrival or trade}}} \Bigg\},$$

where $\underline{O}^S(k) := \mathbb{E}\left[O^S(v) \mid v \le k\right]$.

The coefficient on $\dot K$ must be equal to 0 (the seller is indifferent between speeds of trade):

- If the coefficient on $\dot K$ is negative, then $\dot K = -\infty$: incompatible with smooth trade;
- If the coefficient on $\dot K$ is positive, then $\dot K = 0$: cannot happen after trade begins.

Seller's Problem

Setting the coefficient on $\dot{K}$ equal to 0 in

$$rS(k,\mu) = \sup_{\dot{K}\in(-\infty,0]} \left\{ \mu\lambda\left[\underline{O}^S(k) - S(k,\mu)\right] + \left[P(k,\mu) - S(k,\mu)\right]\frac{f(k)}{F(k)}(-\dot{K}) + \frac{\partial}{\partial k}S(k,\mu)\dot{K} + \frac{\partial}{\partial \mu}S(k,\mu)\dot{\mu} \right\},$$

we obtain the PDEs describing the seller's best response:

$$\frac{\partial}{\partial k}S(k,\mu) = \left[P(k,\mu) - S(k,\mu)\right]\frac{f(k)}{F(k)},$$
$$\frac{\partial}{\partial \mu}S(k,\mu)\dot{\mu} = (\mu\lambda + r)S(k,\mu) - \mu\lambda\underline{O}^S(k).$$

Seller's Problem

$$\frac{\partial}{\partial k}S(k,\mu) = \left[P(k,\mu) - S(k,\mu)\right]\frac{f(k)}{F(k)}, \tag{1}$$

$$\frac{\partial}{\partial \mu}S(k,\mu)\dot{\mu} = (\mu\lambda + r)S(k,\mu) - \mu\lambda \underline{O}^S(k). \tag{2}$$

- The unique solution to the PDE in (2) which is bounded as $\mu \to 0$ gives the **seller's payoff in closed-form**

$$S(k,\mu) = \frac{\mu\lambda}{\lambda + r}\underline{O}^S(k). \tag{3}$$

- Moreover, since

$$(1) \Longleftrightarrow P(k,\mu) = \frac{\frac{\partial}{\partial k}[S(k,\mu)F(k)]}{f(k)}, \tag{4}$$

by (3) and (4), we obtain **prices in closed-form**,

$$P(k,\mu) = \frac{\mu\lambda}{\lambda + r}O^S(k),$$

and that $P(\cdot,\cdot)$ satisfies the PDE

$$\frac{\partial}{\partial \mu}\boldsymbol{P(k,\mu)\dot{\mu}} = \boldsymbol{(\mu\lambda + r)P(k,\mu) - \mu\lambda O^S(k)}.$$

Prices and Seller's Payoffs

Proposition (**Prices and Seller's Payoffs: No Gap Case**)

At any (k, μ) with smooth trade:

- The price is
$$P(k,\mu) = \frac{\mu\lambda}{\lambda + r} O^S(k);$$
- The seller's payoff is
$$S(k,\mu) = \frac{\mu\lambda}{\lambda + r} \underline{O}^S(k).$$

The seller prices competitively

- $P(k, \mu)$ = expected present value the seller would earn from type k by waiting for an arrival;
- That is, each type k pays the opportunity cost of serving him.

$S(k, \mu)$ = expected present value the seller would earn by waiting for an arrival when knowing $v \leq q$.

Buyer's Problem

At state (k, μ) with smooth trade, type $v = k$ buyer's value function must satisfy the HJB equation

$$\underbrace{rB^{v=k}(k,\mu)}_{\substack{\text{expected payoff}\\ \text{in flow terms}}} = \underbrace{\mu\lambda}_{\substack{\text{flow prob.}\\ \text{of arrival}}} \underbrace{\left[O^B(k) - B^{v=k}(k,\mu)\right]}_{\substack{\text{if arrival, game ends, buyer}\\ \text{gets } O^B(k) \text{ and forgoes } B^k}} + \underbrace{\frac{\partial}{\partial k}B^{v=k}(k,\mu)\dot{K} + \frac{\partial}{\partial \mu}B^{v=k}(k,\mu)\dot{\mu}}_{\substack{\text{change in continuation payoff}\\ \text{if no arrival}}}. \tag{5}$$

- Taking the total derivative of the equilibrium condition (reservation price strategies)

$$B^{v=k}(k,\mu) = k - P(k,\mu) \tag{6}$$

yields

$$\frac{\partial}{\partial k}B^{v=k}(k,\mu)\dot{K} + \frac{\partial}{\partial \mu}B^{k=v}(k,\mu)\dot{\mu} = -\frac{\partial}{\partial k}P(k,\mu)\dot{K} - \frac{\partial}{\partial \mu}P(k,\mu)\dot{\mu}. \tag{7}$$

- Finally, replace (6) and (7) into (5) to obtain the PDE describing the buyer's best response

$$\frac{\partial}{\partial k}\boldsymbol{P(k,\mu)\dot{K}} + \frac{\partial}{\partial \mu}\boldsymbol{P(k,\mu)\dot{\mu}} = \boldsymbol{(\mu\lambda + r)P(k,\mu) - (\mu\lambda + r)k + \mu\lambda O^B(k)}.$$

Best Responses

The two players' best responses with smooth trade are described by

$$\frac{\partial}{\partial \mu}P(k,\mu)\dot{\mu} = (\mu\lambda + r)P(k,\mu) - \mu\lambda O^S(k), \tag{8}$$

$$\frac{\partial}{\partial k}P(k,\mu)\dot{K} + \frac{\partial}{\partial \mu}P(k,\mu)\dot{\mu} = (\mu\lambda + r)P(k,\mu) - (\mu\lambda + r)k + \mu\lambda O^B(k). \tag{9}$$

Moreover,

$$P(k,\mu) = \frac{\mu\lambda}{\lambda + r}O^S(k). \tag{10}$$

The **buyer type trading at belief** μ satisfies

$$k = \frac{\mu\lambda O(k)}{\mu\lambda + r} - \frac{\frac{\partial}{\partial k}P(k,\mu)\dot{K}}{\mu\lambda + r} = \begin{cases} \dfrac{\mu\lambda O(k)}{\mu\lambda + r} & \text{with IPV} \\ \dfrac{\mu\lambda O(k)}{\mu\lambda + r} - \dfrac{\mu\lambda O^{S\prime}(k)\dot{K}}{(\lambda + r)(\mu\lambda + r)} & \text{with IV} \end{cases},$$

where: the first equality follows by (8) and (9); the second equality follows by (10).

Trade Dynamics with IPV

With IPV, the buyer type that trades at belief μ satisfies

$$k = \frac{\mu\lambda O(k)}{\mu\lambda + r} \qquad \text{or, equivalently,} \qquad \overbrace{\underbrace{\mu\lambda[O(k) - k]}_{\substack{\text{benefit of}\\ \text{experimentation}}} = \underbrace{rk}_{\substack{\text{cost of}\\ \text{experimentation}}}}^{\text{first-best efficiency condition}}.$$

Theorem (Efficient Trade with IPV)

With independent private values, trade is bilaterally efficient.

Trade Dynamics with IV

With IV, the buyer type that trades at belief μ satisfies

$$k = \underbrace{\frac{\mu\lambda O(k)}{\mu\lambda + r}}_{\text{efficiency condition}} \underbrace{- \frac{\mu\lambda O^{S\prime}(k)\dot{K}}{(\lambda + r)(\mu\lambda + r)}}_{\text{inefficiency}}, \qquad \text{where} \qquad \dot{K} < 0.$$

Theorem (**Inefficient Trade with IV**)

With interdependent values, trade is bilaterally inefficient:

- If $O^{S\prime}(k) > 0$, trade is inefficiently slow (inefficient delay);
- If $O^{S\prime}(k) < 0$, trade is inefficiently fast (inefficient hurry).

Efficiency *vs* Inefficiency

With IPV, the seller's outside option (or opportunity cost of trading with type k),

$$\frac{\mu\lambda}{\lambda + r} O^S,$$

does not depend on the buyer's type.

- Today's self of the seller competes against his future selves;
- Today's self choices do not affect future selves' opportunity costs, which evolve exogenously;
- **No intrapersonal externalities**;
- Trade is efficient.

Alternatively, think of the parallel with competitive equilibrium

- Without externalities, the First Welfare Theorem applies.

Efficiency *vs* Inefficiency

With IV, the seller's outside option (or opportunity cost of trading with type k),

$$\frac{\mu\lambda}{\lambda + r} O^S(k).$$

depends on the the buyer's type k.

- Today's self choices change future selves' opportunity costs;
- **Intrapersonal externalities**;
- Today's self does not internalize changes in future selves' opportunity costs

  Reduction, if ${O^S}'(k) > 0$, or increase, if ${O^S}'(k) < 0$, of the endogenous component of such costs;
- Trade is inefficient.

Trade Dynamics

With IPV, the buyer type that trades at belief μ satisfies

$$\mu\lambda[O(k) - k] = rk.$$

- **Trade begins** either with an **atom** or after a **silent period** (generically)

  At $t = 0$, trade with all types k for which $\mu^0\lambda[O(k) - k] \leq rk$.

- After trade begins, **smooth trade**.
- The market does not clear in finite time.

With IV, similar patterns of trade, but less straightforward to determine which types that trade at $t = 0$.

Discussion

Search and learning as natural rationale for **rich bargaining dynamics**

- Impasse, atoms of trade, delay, long disputes, and slow screening of private information.

Delay is always present but need not be at odds with efficiency

- The **force toward efficiency** may still operate in markets with search and learning.

Efficiency *vs* inefficiency depends on IPV *vs* IV

- Efficient delay with IPV is a knife-edge case;
- **Excessive delay** and **excessive hurry** are both possible as inefficiencies;
- Link type of inefficiency to **sign of the intrapersonal externality among seller's multiple selves**.

Non-trivial efficiency benchmark necessary for identifying both types of inefficiencies.

Discussion

Fuchs and Skrzypacz (2010) and Inderst (2008)

- Outside options are known to exist, but arrive stochastically;
- Common knowledge of gains from trade at $t = 0$, efficiency calls for immediate agreement;
- Immediate agreement with IPV, dynamics with IV (inefficient delay).

Board and Pycia (2014)

- The buyer has an outside option since the beginning of the bargaining process;
- Immediate trade at the static monopoly price (inefficient).

This paper

- Different rationale for delay and trade dynamics
  - Search and learning (uncertainty about whether and when superior outside options exist)
  - Not IV, adverse selection, or lemon problems.
- The source of inefficiency is the same, but the type of inefficiency is different
  - Delay in bargaining not only can be efficient, but it can also be too little;
  - Non-trivial efficiency benchmark is crucial.

Discussion

Ortner (2017)

- Durable goods monopoly with stochastic costs (geometric Brownian motion);
- IPV;
- Delay is efficient with a continuum of types, excessive with discrete valuations;
- Inefficiencies come from market power and rent extraction.

This paper

- Different type of inefficiency
  - Not only efficient delay and excessive delay, but also excessive hurry.
- Different reason for inefficiencies
  - IV (or endogenous part of opportunity cost of trading), not market power or rent extraction;
  - Some rent extraction can come without inefficiencies (next slides).

Market Clearing, Price Dynamics, and Payoffs

Model

Gap case

- The buyer has private valuation $v \in [\underline{v}, 1]$ for the good;
- Here, $0 < \underline{v} < 1$.

Independent private values (IPV)

- $O^S(v) = O^S$ for all $v \in [\underline{v}, 1]$ and some $O^S \geq 0$.

The model remains otherwise unchanged.

Market Clearing

Trade remains efficient with IPV: the buyer type(s) trading at belief μ satisf(y)ies

$$\mu\lambda[O(k) - k] = rk \qquad \text{or, equivalently,} \qquad \frac{\mu\lambda}{\mu\lambda + r} = \frac{k}{O(k)}.$$

Trade stops at time $T_{\underline{v}}$ such that

$$\frac{\mu_{T_{\underline{v}}}\lambda}{\mu_{T_{\underline{v}}}\lambda + r} = \frac{\underline{v}}{O(\underline{v})}, \qquad \text{and so} \qquad T_{\underline{v}} < +\infty \Longleftrightarrow \underline{v} > 0.$$

- **Market clearing** in finite time iff gains from trade become common knowledge in finite time.
- Market clearing **when gains from trade become common knowledge**.

Market Clearing, Price Dynamics, and Payoffs

At any (k, μ) with smooth trade, the price and the seller's payoff are

$$P(k,\mu) = S(k,\mu) = \underbrace{\frac{\mu\lambda}{\lambda + r} O^S}_{\substack{\text{opportunity cost/} \\ \text{pdv outside option}}} + \underbrace{\mathcal{C}(k) \frac{(1-\mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}}}_{\substack{\text{markup/} \\ \text{profit margin}}}.$$

At time $T_{\underline{v}} < +\infty$ the seller clears the market at a positive price by charging the last remaining type $\underline{v}$ his wtp,

$$P(\underline{v}, \mu_{T_{\underline{v}}}) = \underline{v} - \frac{\mu_{T_{\underline{v}}}\lambda}{\lambda + r} O^B(\underline{v}) > 0,$$

which is used as terminal condition to determine $\mathcal{C}(k)$.

Market Clearing, Price Dynamics, and Payoffs

Proposition (**Prices and Seller's Payoffs: Gap Case**)

At any (k,μ) with smooth trade, the price and the seller's payoff are

$$P(k,\mu) = S(k,\mu) = \underbrace{\frac{\mu\lambda O^S}{\lambda+r}}_{\substack{\text{opportunity cost/}\\ \text{pdv outside option}}} + \underbrace{\left(\frac{\mu_{T_{\underline{v}}}}{\mu}\right)^{r/\lambda}\left(\frac{1-\mu}{1-\mu_{T_{\underline{v}}}}\right)^{(\lambda+r)/\lambda}\left(\underline{v} - \frac{\mu_{T_{\underline{v}}}\lambda}{\lambda+r}O^B(\underline{v}) - \frac{\mu_{T_{\underline{v}}}\lambda}{\lambda+r}O^S\right)}_{\substack{\text{markup/}\\ \text{profit margin}}}.$$

- Prices decrease over time in a seller's ($O^B(k) = 0$ for all $v \in [\underline{v}, 1]$);
- Prices increase over time in a buyer's market ($O^S = 0$ for all $v \in [\underline{v}, 1]$);
- Prices need not be monotone in a general market.

Market Clearing, Price Dynamics, and Payoffs

Whether prices and payoffs are higher than competitive depends on a **market clearing condition**.

Proposition (**Market Clearing and Prices**)

(i) The market clears in finite time if and only if the market clearing price is positive.

(ii) If the market clearing price is positive
- Prices and the seller's payoff are higher than the competitive ones.

(iii) If the market does not clear in finite time, the seller trades at his opportunity cost.

Bargaining Externalities and Market Unraveling

Bargaining Externalities and Market Unraveling

So far, focus on bilateral surplus

- Consistent with viewing bargaining relationships as bilateral monopolies.

Suppose the event corresponds to the arrival of new traders

- Original players do not internalize how their agreements' timing affects new traders' payoffs;
- **Bargaining externality**.

Bargaining Externalities and Market Unraveling

Arrival of new seller with an upgraded good (buyer's valuation for original good drops to 0).

- Buyer's private valuation for the upgraded good: $\tilde{v} \sim U[0,1]$ (unknown before arrival)
- The new seller has commitment power and sets $p^* = 1/2 = \arg\max_p p(1-p)$, so that

$$O^S(v) = 0, \qquad O^B(v) = E(\tilde{v} \mid \tilde{v} \geq 1/2) - p^* = 1/4, \qquad O^S_{\text{new}}(v) = 1/4.$$

Freestyle bargaining is bilaterally efficient (IPV), but *not* efficient

$$\underbrace{v = \frac{\mu\lambda}{4(\mu\lambda + r)}}_{\substack{\text{freestyle bargaining} \\ \text{bilateral efficiency}}} \qquad vs \qquad \underbrace{v = \frac{\mu\lambda}{2(\mu\lambda + r)}}_{\text{efficiency}}.$$

- From the viewpoint of total surplus, trade under freestyle bargaining is too fast.
- Coasean forces/seller's lack of commitment leads to substantial welfare losses.

Bargaining Externalities and Market Unraveling

Suppose the original seller has commitment power

- The seller screens buyer types at a slower rate
- In particular, if $v \sim U[0,1]$,

$$v = \frac{\mu\lambda}{2(\mu\lambda + r)}.$$

  That is, trade is efficient, but too slow from the viewpoint of bilateral surplus.

Proposition (Bargaining Externalities and Market Unraveling)

Standard welfare implications of the Coase conjecture can be overturned

- The commitment solution can outperform the no commitment solution in terms of total surplus.

Conclusion

Conclusion

Mechanism Design

- Is the first best implementable with IV? Is it so in prices?
- Is bargaining with IV second-best efficient?
- Do inefficiencies (and their typology) depend on the chosen extensive form?
- Seller has commitment power.

Equilibrium Notion

- What matters beyond stationarity?
- The equilibrium notion seems “conservative” with IPV;
- Largest class with equilibrium existence and uniqueness?

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